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CONCERNING INFINITY

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My thesis, which I shall explain, and in outline defend, is this: the sum total of all that exists, the 'universe' is in no way infinite. In particular, it contains neither the infinitely large nor the infinitely small. That being so, why do we ever think about infinity, or concern ourselves with it? Because we worry about the future, and its infinite possibilities, but the future is not part of the universe, as it is. To say that the future will exist is a tautology. To say that the future does exist is an error.

The usefulness, for a philosopher, in showing that there can be no infinity, is that the concept of infinity leads on to paradox in a number of different ways, despite the optimistic hopes of Cantor and Russell that this might be avoided; and no logical thinker can be happy in a world that contains paradox. My proof of the contingent fact that the universe is wholly finite, does not, of course, eliminate the logical fact that the concept of intinity involves the logician in paradox, but it greatly diminishes its importance. This is especially true if one regards logic, not as an 'absolute', but as a gift, given to us human beings, solely for the purpose of investigating this universe. Perhaps some higher intellectual power, to which we do not have access, would be needed to cope with a counter-factual intinite world.

My proof is based on the same premises as Empiricism. Rationalist philosophers are entitled therefore to reject it. It is also 'Foundationalist', in that it assumes that we have, as the foundation ot our knowledge, a finite array of particular truths, whlch are certain; and that we can explore these by some form of logic, to reveal the probable general truths, which the particular certainties imply.

For the classical empiricist philosophers, the finite array of certain particular truths were facts of observation, and the logic was classical induction. In a typical investigation, some of the available observations of a particular kind are taken as a representative sample of all the facts of that kind that the universe contains; and if all the members of the sample have a particular property, or comply with a particular theory, then it is provisionally supposed that all the other facts in the universe do so too. Betore any such induction is fully accepted, a number ot questions are asked. The two that concern us here are:

1. Was the sample big enough and random enough to be regarded as representative?

2. Is the theory the simplest available that will work?

If the number of relevant facts in the universe were intinite, no finite number of observations could ever be regarded as a representative sample.

A simple way of seeing that this is so, is to attempt to apply inductive techniques to numbers, of which there are an infinite number, and whose propertles are known in advance by the deductive science of arithmetic; and checking whether induction gives the same result as deduction. Induction frequently fails this test (l). This is because however one selects one's sample, it always suffers from the bias that the numbers chosen are much nearer to naught than they are to infinity. The same kind ot difficulty wlll always arise in sampling any infinite universe.

The need for the second question, 'Is the theory simple?' implies, if you look at the matter carefully, that there is no place for infinity in a universe which is suitable for inductive logic. In such a universe where can this infinity be? It cannot be in the finite array of observed experiences, so it must be ln the theories that have been, or may in the future be thought up in order to explain them. It is a requirement of both of these classes of theory that they should be simple. Now if any of them contained an infinity of any kind, they would, as a consequence of that fact, not only be complex, but infinitely complex. It follows that there is nowhere in our universe for infinity to lurk, and so we can conclude that. it is't there. I suggest that no theory at all, just a list of all the many experiences and a statement that they are there and un-explained; is simpler than any theory that contains infinity.

At first sight this may seem to conflict with what actually seems to occur when inductive logic is used, as in science. The theories that scientist actually use sometimes do seem to contain, or at least to allow an intinity. If a theory containing intfnity is really out of bounds, how do they get away with it?

Scientists are pragmatic, and their use for a theory is as an instrument for accurate prediction and for technological invention. They are unconcerned with questions of ultimate reality. If their theory works for these practical purposes, and if, for these purposes the apparent infinlty makes no difference, these pragmatic people see no need to eliminate it, especially as the verbal or algebraic expression of the theory is often shorter if the infinity is not eliminated, though this does not imply that the theory is simpler. This general claim could only be conclusively proved by probing every scientific theory that ever was, but a discussion of one very familiar one will show how reasonable it is. I shall discuss the concept of space as employed in Newtonian physics. The fact that this is not up to date science is un-important, for I am discussing the way scientific theories are used, not whether they are true.

Newtonian space extended for ever in all directions, and presumably contained an infinite number of stars. These infinite features don't matter for the science, for all the scientific results would be exactly the same if the Newtonian universe were a 'wallpaper' universe. I shall next explain what I mean by a 'wallpaper' universe. Wallpaper is of course being used as a metaphor, and the feature ot wallpaper of importance in the metaphor is the pattern which is printed on it. The pattern on a wall-paper repeats itself exactly, in each of its two dimensions, and it your piece of wallpaper were big enough, the repeats would go on for ever. All the things in what I am calling a 'wallpaper' Newtonian universe are like this, except, of course, that it has three dimensions. Take a particular bit of the pattern, say a particular formal flower, unique in the pattern itself. How many flowers are there altogether? Lots of them, if you are looking from in front of the wall. If however the infinite sheet of wallpaper were the whole universe, there would be no 'in front' to look at it from. What dlfference does this make? A great deal lf you accept Leibniz' prlnciple of the identity of indiscernlbles. According to this princlple, it two things are absolutely lientical in every respect, including their spatial and other relationships to everything else that can be 'discerned' in the whole unlierse, they are identical, and there is really only one of them. By this principle there is only one flower on all our infinite wallpaper, and only one edition ot your good self, gentle reader, in my 'wallpaper' universe; though you, the earth, the sun, and the rest are repeated an infinite number ot times in each direction, at, of course immense distances. Perhaps if you got a sufficiently powerful telescope you would see yourself in the distance, looklng down a telescope at an even further edition of yourself; but if you could do it, this would not prove that there were two or more of you, any more than an experiment with mirrors could prove the same thing.

So much for infinitely large distances. Infinite time can be dealt with by supposing it cyclic.

What however, about the infinitely small? Surely science supposes that distance or time can be subdivided indetinitely? No, not really. The mathematlcians do so, and have tremendous fun as a result, but we philosophers do not have to worry about them. We are interested in truth, they merely in mischief. As far as science is concerned, the task is to accommodate finite data. Every measurement is only accurate to a finite extent, and even our remote descendants, whose instruments are likely to be superlatively good, will only be much more accurate than us, never infinitely so, for they will always be finite beings. The theories of science will always therefore be consistent with the view that there is some quantum of length or distance, which is so short that it is impossible to divide it. They will of course never scientifically prove it, or provide any evidence in favour of it either. If they did it would cease to be the metaphysical theory which I am suggested, but would play a part in the theories of science, and if it ever did that, the scientists would probably proceed to split it, like they did the atom that Dalton gave them. To prevent such problems we must imagine the metaphysical quantum of length to be so small, that its existence will never affect, and therefore be safe from the measurements of any future scientist.

If these ideas of a wallpaper universe, and of a quantum of length seem unnecessary complications, consider how complicated infinity may be. Perhaps each atom, electron, quark, or whatever happens to be in fashion this week as the smallest particle known to science, is in fact a whole miniature universe, and each of its ultimate particles is another universe, and so on? Why not? Infinity never comes to an end. In the face of that sort of complexity, my finite suggestions seem very simple.

I would like to conclude this short article by noting the simplification in our thinking that getting quite rid of infinity achieves. Firstly irrational numbers go, and good riddance. For an example to show how this works, take π, the ratio of the circumference of a circle to its diameter. Draw the biggest possible circle without going outside Einstein's hyper-sphere, or outside our wallpaper pattern. Measure its diameter and circumference in quanta of length. You get two big integral numbers, and their ratio is a vulgar fraction! All lesser examples of π will be even more vulgar. The Differential Calculus remains with us however, and, as a bonus, actually becomes logical at last. The bland, and quite un-justified assurance that we know what will happen in the limit as ∂x approaches zero, is replaced with the much more reasonable conviction that we know well enough approximately what will happen when it gets as small as is necessary, for whatever problem in applied mathematics we have in hand.

The real bonus is however the elimination of paradox. Most of the paradoxes, both those that amused the Greeks, and the new ones that have intrigued the twentieth century, obviously involve infinity. One that does not very obviously do so is Russell's amusing paradox about sets or classes. More particularly it was concerned with classes of all classes that were, or were not, members of their own class. We do not need here to rehearse the anomaly, which has been frequently discussed (2). The point that concerns us is that, even in a universe which is in all respects finite, indeed in a universe that contains only one entity in it, there are an infinite number of possible classes, once one allows a class to be a member of a class. Russell's famous paradox is indeed concerned with possible classes of classes! We therefore do not need to worry about it any more.

1. See Bertrand Russell *Human Knowledge its Scope and Limits*, 1948 p419(2).

2. For a simple statement see Russell's *My Philosophical Development* p74..